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William C. Moss

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William C. Moss

University of California, Lawrence Livermore National Laboratory

Livermore, California 94550

Abstract

The dependence of the strain rate on the Hugoniot stress and the effective viscosity is derived for steady wave propagation, by considering the balance between viscous and driving forces in the steady wave. The strain rate in elastic-plastic solids is shown to vary as the fourth power of the Hugoniot stress, which is in agreement with experimental data.

Advances in time resolved measurements of shock wave profiles have prompted many theoretical and experimental investigations of shock wave structure. The occurrence of steady waves (time independent wave profiles) in solids, first observed by Johnson and Barker[1], has yielded information on the relationship between the strain rate and effective viscosity in the shock front. Grady[2] obtained the interesting theoretical result that the maximum value of the strain rate in an elastic-plastic steady wave is proportional to σ_H^4, where σ_H is the Hugoniot stress. This result agreed with experimental data for aluminum.[3] A compilation of more recent experimental data for many solid materials[4] shows the same quartic dependence of strain rate on Hugoniot stress.

Although Grady's theoretical result is in agreement with experimental data, the derivation relies on the postulated invariance of a quantity called the "shock adiabatic invariant". In this correspondence, we present a general derivation for the strain rate dependence of the Hugoniot stress, for steady wave compression of solids, liquids, and gases, by considering the balance between viscous and driving forces in the steady wave. The "shock adiabatic invariant" is not needed. In particular, we show that $\dot{e} \propto \sigma_H^4$, for elastic-plastic solids, by using well known relations from plasticity theory.

A state of stress (σ_{ij}) can always be divided into isotropic $(P\delta_{ij})$ and deviatoric (σ'_{ij}) components:

$$\sigma_{ij} = \sigma'_{ij} + P\delta_{ij}, \tag{1}$$

where δ_{ij} is the Kronecker delta, and the pressure P is the mean stress $(\sigma_{kk}/3)$. We use the convention that compressive stresses are positive. We are interested in one-dimensional waves, so we write

$$\sigma = \sigma' + P, \tag{2}$$

where σ and σ' represent components of stress in the direction of wave propagation, that is, longitudinal stresses. We assume that the pressure can be represented by a Mie-Grüneisen equation. We write[5]

$$P(e,E) = -\sigma'_H(e) + \frac{\rho_o c^2 e}{(1-se)^2}\left(1 - \frac{\gamma_o e}{2}\right) + \rho_o \gamma_o E, \tag{3}$$

where $e = 1 - \rho_o/\rho$ is the longitudinal strain (which is identical to the volumetric strain for 1-d loading), E is the specific internal energy, ρ_o is the reference density of the material, c is the bulk sound speed at the reference density and temperature (300K), s is the slope of the shock velocity-particle velocity relation, which is assumed to be linear, and the volume dependence of the Grüneisen gamma γ is assumed to be $\rho\gamma = \rho_o\gamma_o$. $\sigma'_H(e)$ is the longitudinal deviatoric stress along the Hugoniot. (Equation (3) reduces correctly to $P_H = -\sigma'_H + \sigma_H$, for Hugoniot states.) We have ignored the effects of the Hugoniot-Elastic-Limit and the elastic deviatoric specific internal energy, which are negligible, in Eq. (3).[5,6] Substituting Eq. (3) into Eq. (2), we obtain:

$$\sigma = \sigma' - \sigma'_H(e) + \frac{\rho_o c^2 e}{(1-se)^2}\left(1 - \frac{\gamma_o e}{2}\right) + \rho_o\gamma_o E. \tag{4}$$

We are interested in the effect of viscosity on the time dependence of σ. In general, there are shear and bulk viscosities, however, by choosing a Mie-Grüneisen equation, we have neglected the bulk viscosity. Thus, Eq. (4) shows that the time dependence of σ is due to the time dependence of σ', and we regard $\sigma' - \sigma'_H(e)$ as a viscous stress. We write

$$\sigma'_{\text{vis}} \equiv \eta\dot{e} = \sigma' - \sigma'_H(e), \tag{5}$$

where η is the viscosity. This definition of σ'_{vis} satisfies the necessary boundary conditions: Before the shock arrives, $\sigma' = \sigma'_H(e=0) = 0$ and $\dot{e} = 0$. After the shock passes, $\sigma' = \sigma'_H(e_H)$ and $\dot{e} = 0$. During the shock, $\sigma' \neq \sigma'_H(e)$, because the Hugoniot is not a thermodynamic path, thus, $\dot{e} \neq 0$.

It is well known that as the strain rate increases, the shock rise time decreases. We let the viscosity reflect this experimental observation. We write

$$\eta \equiv \delta\dot{e}^{-\alpha}, \qquad 0 < \alpha < 1 \tag{6a}$$

where δ is a constant. We rewrite Eq. (6a) using Eq. (5):

$$\sigma'_{\text{vis}} = \delta\dot{e}^{1-\alpha}. \tag{6b}$$

α must be less than one, so that $\sigma' \to 0$, as $\dot{e} \to 0$. α must be greater than zero, so that the viscosity decreases, as the strain rate increases. ($\alpha = 0$ corresponds to Newtonian viscosity.)

When steady shocks are present, the longitudinal stress must lie on the Rayleigh line (σ_R). We write

$$\sigma_R \equiv \frac{\sigma_H e}{e_H} = \frac{\rho_o c^2 e}{(1 - s e_H)^2}, \tag{7}$$

where σ_H is obtained by evaluating Eq. (3) along the Hugoniot. Combining Eqs. (4), (6b), and (7), we obtain

$$\frac{\rho_o c^2 e}{(1 - s e_H)^2} = \delta \dot{e}^{1-\alpha} + \frac{\rho_o c^2 e}{(1 - s e)^2}\left(1 - \frac{\gamma_o e}{2}\right) + \rho_o \gamma_o E_R, \tag{8}$$

where E_R is the specific internal energy along the Rayleigh line. Using Eq. (7), we obtain

$$E_R = -\int_{V_o}^{V} \frac{\sigma_H e}{e_H}\, dV = \frac{V_o \sigma_H}{e_H} \int_0^e x\, dx = \frac{V_o \sigma_H e^2}{2 e_H}. \tag{9}$$

Substituting into Eq. (8) yields

$$e = \left(\frac{\rho_o c^2 e}{\delta}\left((1 - s e_H)^{-2} - (1 - s e)^{-2}\right)\left(1 - \frac{\gamma_o e}{2}\right)\right)^{\frac{1}{1-\alpha}}. \tag{10}$$

When α and δ are specified, Eq. (10) gives the strain rate at all points in the steady wave ($0 \le e \le e_H$).

Equation (10) can be approximated by an expansion to lowest order in e, when σ_H is small ($\sigma_H \le 20\,\mathrm{GPa}$). We write

$$\dot{e} \approx \left(2 s \rho_o c^2 \delta^{-1} e (e_H - e)\right)^{\frac{1}{1-\alpha}}. \tag{11}$$

We take the maximum value of the strain rate ($\dot{e}_m$), and express it to lowest order in σ_H. We obtain

$$\dot{e}_m = \left(\frac{s \rho_o c^2 e_H^2}{2\delta}\right)^{\frac{1}{1-\alpha}} = \left(\frac{s \sigma_H^2}{2 \delta \rho_o c^2}\right)^{\frac{1}{1-\alpha}}. \tag{12}$$

In solids, the viscosity can be attributed to plasticity. Consequently, we assume that the total strain rate can be divided into elastic and plastic components, thus Eq. (6b) can be rewritten as

$$\sigma'_{\text{vis}} = \delta \dot{e}^{1-\alpha} = \delta(\dot{e}^e + \dot{e}^p)^{1-\alpha} \approx \delta(\dot{e}^p)^{1-\alpha}, \tag{13}$$

where we have assumed that the total strain rate is mostly plastic. We may obtain a value for α using a plasticity relationship postulated by Taylor[7]

$$\sigma' \propto \Lambda^{0.5}, \tag{14a}$$

and the Orowan relation

$$\dot{e}^p = \Lambda v b, \tag{14b}$$

where Λ is the dislocation density, v is the dislocation velocity, and b is the Burgers vector. Equations (14a,b) imply that

$$\sigma' \propto (\dot{e}^p)^{0.5}. \tag{15}$$

Comparing Eqs. (13) and (15), we obtain

$$\alpha = 0.5. \tag{16}$$

Substituting Eq. (16) into Eq. (12) yields

$$\dot{e} = \left(\frac{s}{2\delta\rho_o c^2}\right)^2 \sigma_H^4. \tag{17}$$

Equation (17) predicts the same functional dependence noted by Grady for a variety of materials[4] [Be, U, Fe, MgO, SiO_2, Al, Cu, and Bi].[8]

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References

[1] J. N. Johnson and L. M. Barker, J. Appl. Phys. , **40**, 4321 (1969).

[2] D. E. Grady, Appl. Phys. Lett. , **38**, 826 (1981).

[3] L. M. Barker in *Behavior of Dense Media Under High Dynamic Pressures* (Gordon and Breach, New York, 1968). p. 483.

[4] D. E. Grady, J. Appl. Phys. , in press.

[5] W. C. Moss, J. Appl. Phys. , **55**, 2741 (1984).

[6] C. E. Morris and J. N. Fritz, J. Appl. Phys. , **51**, 1244 (1980).

[7] G. I. Taylor, Proc. Roy. Soc. London, **A145**, 362 (1934).

[8] The stress regime of the data presented by Grady (ref. [4]) are $\leq$15 GPa , so that our expansion to lowest order in e is valid.